|  | Sheet 2  Chapter One |  |  |
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1. **Let P(x) be the statement “x spends more than five hours every weekday in class,” where the domain for x consists of all students. Express each of these quantifications in English.**
   1. **∃xP(x)**
   2. **∀xP(x)**
   3. **∃x ¬P(x)**
   4. **∀x ¬P(x)**
2. **Translate these statements into English, where C(x) is “x is a comedian” and F(x) is “x is funny” and the domain consists of all people.**
   1. **∀x(C(x) → F(x))**
   2. **∀x(C(x) ∧ F(x))**
   3. **∃x(C(x) → F(x))**
   4. **d) ∃x(C(x) ∧ F(x))**
3. **Let P(x) be the statement “x can speak Russian” and let Q(x) be the statement “x knows the computer language C++.” Express each of these sentences in terms of P(x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.**
   1. **There is a student at your school who can speak Russian and who knows C++.**
   2. **There is a student at your school who can speak Russian. but who doesn’t know C++.**
   3. **Every student at your school either can speak Russian or knows C++.**
   4. **No student at your school can speak Russian or knows C++.**
4. **Determine the truth value of each of these statements if the domain consists of all integers.**
   1. **∀n(n + 1 > n)**
   2. **∃n(2n = 3n)**
   3. **∃n(n = −n)**
   4. **∀n(3n ≤ 4n)**
5. **Suppose that the domain of the propositional function P(x) consists of the integers 0, 1, 2, 3, and 4. Write out each of these propositions using disjunctions, conjunctions, and negations.**
   1. **∃xP(x)**
   2. **∀xP(x)**
   3. **∃x¬P(x)**
   4. **∀x¬P(x)**
   5. **¬∃xP(x)**
   6. **¬∀xP(x)**
6. **Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.**
   1. **Someone in your class can speak Hindi.**
   2. **Everyone in your class is friendly.**
   3. **There is a person in your class who was not born in California.**
   4. **A student in your class has been in a movie.**
   5. **No student in your class has taken a course in logic programming.**
7. **Express each of these statements using logical operators, predicates, and quantifiers.**
   1. **Some propositions are tautologies.**
   2. **The negation of a contradiction is a tautology.**
   3. **The disjunction of two contingencies can be a tautology.**
   4. **The conjunction of two tautologies is a tautology.**
8. **Translate these specifications into English where F(p) is “Printer p is out of service,” B(p) is “Printer p is busy,” L(j) is “Print job j is lost,” and Q(j) is “Print job j is queued.”**
   1. **∃p(F(p) ∧ B(p)) → ∃jL(j)**
   2. **∀pB(p) → ∃jQ(j)**
   3. **∃j (Q(j) ∧ L(j)) → ∃pF(p)**
   4. **(∀pB(p) ∧ ∀jQ(j)) → ∃jL(j)**
9. **What are the truth values of these statements?**
   1. **∃!xP(x) → ∃xP(x)**
   2. **∀xP(x) → ∃!xP(x)**
   3. **∃!x¬P(x)→¬∀xP(x)**
10. **Express each of these mathematical statements using predicates, quantifiers, logical connectives, and mathematical operators.**
    1. **The product of two negative real numbers is positive.**
    2. **The difference of a real number and itself is zero.**
    3. **Every positive real number has exactly two square roots**
    4. **A negative real number does not have a square root that is a real number.**
11. **Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.**
    1. **∃x∀y(xy = y)**
    2. **∀x∀y(((x < 0) ∧ (y < 0)) → (xy > 0))**
    3. **∃x∃y((x2 > y) ∧ (x < y))**
    4. **∀x∀y∃z(x + y = z)**
12. **Determine the truth value of each of these statements if the domain for all variables consists of all integers.**
    1. **∀n∃m(n2 < m)**
    2. **∃n∀m(n < m2)**
    3. **∀n∃m(n + m = 0)**
    4. **∃n∀m(nm = m)**
    5. **∃n∃m(n2 + m2 = 5)**
    6. **∃n∃m(n2 + m2 = 6)**
    7. **∃n∃m(n + m = 4 ∧ n − m = 1)**
    8. **∃n∃m(n + m = 4 ∧ n − m = 2)**
    9. **∀n∀m∃p(p = (m + n)/2)**
13. **Suppose the domain of the propositional functionP(x, y) consists of pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.** 
    1. **∀x∀yP(x, y)**
    2. **∃x∃yP(x, y)**
    3. **∃x∀yP(x, y)**
    4. **∀y∃xP(x, y)**
14. **Express the negations of each of these statements so that all negation symbols immediately precede predicates.**
    1. **∀x∃y∀zT (x, y, z)**
    2. **∀x∃yP(x, y) ∨ ∀x∃yQ(x, y)**
    3. **∀x∃y(P(x, y) ∧ ∃zR(x, y, z))**
    4. **∀x∃y(P(x, y) → Q(x, y))**
15. **Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).**
    1. **¬∀x∀yP(x, y)**
    2. **¬∀y∃xP(x, y)**
    3. **¬∀y∀x(P(x, y) ∨ Q(x, y))**
    4. **¬(∃x∃y¬P(x, y) ∧ ∀x∀yQ(x, y))**
    5. **¬∀x(∃y∀zP(x, y, z) ∧ ∃z∀yP(x, y, z))**
16. **Determine the truth value of the statement ∀x∃y(xy = 1)if the domain for the variables consists of**
    1. **the nonzero real numbers.**
    2. **the nonzero integers.**
    3. **the positive real numbers**

***Best of luck!***